[4]Hasegawa, S., Echigo, R., and Fukuda, K., "Analytical and Experimental Studies on Simultaneous Radiative and Free Convective Heat Transfer Along a Vertical Plate," *Proceedings of the Japanese Society of Mechanical Engineers*, Vol. 38, 1972, pp. 2873–2882.
[5]Cheng, E. H. and Özişik, M. N., "Radiation with Free Convection in an Absorbing, Emitting, and Scattering Medium," *International Journal of Heat and Mass Transfer*, Vol. 15, June 1972, pp. 1243–1252.
[6]Bankston, J. D., Lloyd, J. R., and Novotny, J. L., "Radiation-Convection Interaction in an Absorbing-Emitting Liquid in Natural Convection Boundary Layer Flow," *ASME Journal of Heat Transfer*, Vol. 99, February 1977, pp. 125–127.
[7]Cess, R. D., "The Interaction of Thermal Radiation with Free Convection Heat Transfer," *International Journal of Heat and Mass Transfer*, Vol. 9, 1966, pp. 1269–1277.
[8]Audunson, T. and Gebhart, B., " An Experimental and Analytical Study of Natural Convection with Appreciable Radiation Effects," *Journal of Fluid Mechanics*, Vol. 52, March 1972, pp. 57–95.
[9]Cess, R. D., "The Interaction of Thermal Radiation with Conduction and Convection Heat Transfer," *Advances in Heat Transfer*, Vol. 1, edited by T. F. Irvine and J. P. Hartnett, Academic Press, New York, 1964, pp. 1–50.
[10]Patankar, S. V. and Spalding, D. B., *Heat and Mass Transfer in Boundary Layers*, 2nd ed., Intertext Books, London, 1970.
[11]Incropera, F. P. and DeWitt. D. P., *Introduction to Heat Transfer*, Wiley, New York, 1985.

Step Heat Flux Effects on Turbulent Boundary-Layer Heat Transfer

Robert P. Taylor,* Philip H. Love,† Hugh W. Coleman,‡ and M. H. Hosni†
Mississippi State University, Mississippi State, Mississippi

Introduction

THIS Note presents heat-transfer data for the case of incompressible turbulent flow of air over a smooth flat plate with an unheated starting length followed by a heated region with a constant wall heat flux. To the authors' knowledge, no experimental data have been reported in the literature for this step heat flux boundary condition. In their definitive work, Reynolds, Kays, and Kline[1] present an integral solution for this case and experimental results for a double-pulse heat flux boundary condition and an arbitrary heat flux boundary condition.

Experiments

A complete description of the facility and its qualification is presented by Coleman et al.[2] This closed-loop air tunnel has a freestream velocity range of 6 to 67 m/s. The air temperature is controlled with a heat exchanger and cooling water loop. A system of honeycomb and screens produces a freestream turbulence intensity at the nozzle exit of less than 0.3%. The thermal boundary condition is set by computer control of the electrical power to each of the 24 individual plates that make up the bottom surface of the nominally 0.1 m high by 0.5 m wide by 2.4 m long test section. Each 0.1 m plate is at a uniform temperature. The top wall of the test section can be adjusted to maintain a constant freestream velocity. The boundary layer is tripped with a 1 mm high by 12 mm wide wooden strip that is located immediately in front of the test surface.

The data reduction expression for the Stanton number is obtained from an energy balance on each test plate as

$$St = \frac{W - q_r - q_c}{A\rho c_p u_\infty (t_w - T_\infty)} \tag{1}$$

Here, W is the electric power to the plate, q_r the radiative heat loss, q_c the conductive heat loss, A the plate area, ρ the air density, c_p the air-specific heat, u_∞ the freestream velocity, t_w the plate temperature, and T_∞ the freesteam stagnation temperature. All properties are evaluated at the freestream static temperature. The details of these measurements and estimation of the uncertainty are given by Love et al.[3] The St uncertainty, U estimates are presented in Tables 1 and 2.

Theories

The classic solution for the step heat input problem was presented by Reynolds, Kays, and Kline.[1] They used the step wall temperature solution of the integral boundary-layer equations as the kernel in a superposition interal to obtain the Stanton number $St(\phi;x)$ for an unheated length ϕ, followed by a constant heat flux as

$$\frac{St_t(x)}{St(\phi;x)} = \frac{\beta_r(1/9,10/9)}{\Gamma(1/9)\Gamma(8/9)} \tag{2}$$

$$r = 1 - (\phi/x)^{0.9}$$

where $St_t(x)$ is the Stanton number for a constant wall temperature without starting length, Γ the gamma function, and β_r the incomplete beta function

$$\beta_r(a,b) = \int_0^r z^{a-1}(1-z)^{b-1}\,dz \tag{3}$$

The results of the experiments are also compared with finite-difference solutions of the partial differential equations of the boundary layer. The solutions presented here are based on a mixing length turbulence model with van Driest damping and a turbulent Prandtl number, $Pr_t = 0.9$. All computations presented were made with the BLACOMP code as verified by Gatlin.[4]

Results

Stanton number measurements were made for six cases ($\phi = 0.3$, 0.7, and 1.3 m at $u_\infty = 28$ m/s and 0.5, 0.8, and 1.3 m at $u_\infty = 67$ m/s). The cases were selected to obtain an appropriate spread in Reynolds numbers, Re_ϕ. The results of these measurements are presented in Tables 1 and 2.

Figure 1 shows a summary of the Stanton number data for a constant heat flux boundary condition and the unheated starting length cases for $u_\infty = 28$ m/s and $u_\infty = 67$ m/s. The figure shows that as the thermal boundary layer develops, the unheated starting length Stanton numbers approach the results for the constant heat flux boundary condition. The first heated plate is highlighted in each case by plotting its data as a solid symbol. Data from the last plate are not plotted for any case. The curve in Fig. 1 is Eq.(2) for $\phi = 0$, $St/St_t = 1.043$, with $St_t = 0.185\, Pr^{-0.4}\, [\log_{10}(Re_x)]^{-2.584}$.[2,3] This St_t expression is the usual $St_t Pr^{0.4} = C_f/2$ with the well-known Schultz–Grunow correlation for C_f.[1]

Received Oct. 21, 1988; revision received Feb. 9, 1989.

*Associate Professor, Thermal & Fluid Dynamics Laboratory, Mechanical and Nuclear Engineering Department. Member AIAA.
†Graduate Research Assistant, Thermal & Fluid Dynamics Laboratory, Mechanical and Nuclear Engineering Department.
‡Professor, Thermal & Fluid Dynamics Laboratory, Mechanical and Nuclear Engineering Department. Member AIAA.

Table 1 Data for a nominal freestream velocity of 28 m/s

		$u_\infty = 27.8$ m/s $T_\infty = 26.8°$C					$u_\infty = 27.9$ m/s $T_\infty = 26.5°$C					$u_\infty = 27.8$ m/s $T_\infty = 26.4°$C					$u_\infty = 27.8$ m/s $T_\infty = 26.3°$C				
P#	$x(m)$	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%
1	0.05	34.3	44.9[a]	0.09	3.95	—[b]	26.5	—	—	—	—	26.3	—	—	—	—	26.2	—	—	—	—
2	0.15	36.3	44.4	0.26	3.09	4.2	27.0	—	—	—	—	26.6	—	—	—	—	26.4	—	—	—	—
3	0.25	37.7	44.8	0.44	2.71	3.6	28.0	—	—	—	—	26.7	—	—	—	—	26.5	—	—	—	—
4	0.36	38.6	44.4	0.62	2.46	3.3	35.1	45.4[a]	0.62	3.45	—[b]	26.9	—	—	—	—	26.7	—	—	—	—
5	0.46	39.5	44.6	0.80	2.31	3.0	37.2	45.3	0.80	2.76	3.4	27.0	—	—	—	—	26.7	—	—	—	—
6	0.56	40.1	44.7	0.97	2.21	2.8	38.5	45.3	0.98	2.46	2.9	27.3	—	—	—	—	26.8	—	—	—	—
7	0.66	40.5	44.3	1.15	2.13	2.7	39.2	45.3	1.15	2.33	2.7	27.9	—	—	—	—	26.9	—	—	—	—
8	0.76	41.1	44.6	1.33	2.05	2.5	40.0	45.4	1.33	2.19	2.5	35.6	44.0[a]	1.33	3.10	—[b]	27.0	—	—	—	—
9	0.86	41.6	44.5	1.50	1.98	2.4	40.5	45.4	1.51	2.11	2.4	37.5	44.2	1.51	2.59	3.6	27.1	—	—	—	—
10	0.97	41.9	44.4	1.68	1.93	2.4	40.9	45.1	1.69	2.03	2.3	38.5	44.2	1.69	2.37	3.2	27.1	—	—	—	—
11	1.07	42.2	44.5	1.86	1.90	2.3	41.3	45.3	1.87	1.99	2.3	39.2	44.1	1.87	2.23	3.0	27.3	—	—	—	—
12	1.17	42.5	44.3	2.03	1.86	2.3	41.7	45.3	2.05	1.94	2.2	39.7	43.7	2.05	2.14	2.9	27.6	—	—	—	—
13	1.27	42.8	44.3	2.21	1.82	2.2	42.1	45.4	2.23	1.90	2.2	40.2	43.9	2.22	2.06	2.7	29.0	—	—	—	—
14	1.37	43.0	44.3	2.39	1.79	2.2	42.4	45.5	2.40	1.86	2.2	40.6	44.3	2.40	2.01	2.6	35.3	44.7[a]	2.40	3.24	—[b]
15	1.47	43.2	44.3	2.56	1.78	2.2	42.6	45.3	2.58	1.83	2.2	41.0	43.7	2.58	1.95	2.5	38.0	44.6	2.58	2.49	3.2
16	1.57	43.5	44.2	2.74	1.74	2.2	42.9	45.2	2.76	1.80	2.2	41.4	44.0	2.76	1.91	2.4	39.2	44.4	2.76	2.24	2.8
17	1.68	43.7	44.7	2.92	1.74	2.2	43.1	45.4	2.94	1.78	2.2	41.6	44.0	2.94	1.88	2.4	39.8	44.4	2.94	2.14	2.6
18	1.78	43.8	44.3	3.09	1.71	2.2	43.2	45.1	3.12	1.76	2.2	41.8	43.9	3.12	1.85	2.3	40.3	44.7	3.11	2.08	2.5
19	1.88	44.0	44.3	3.27	1.69	2.2	43.4	45.2	3.30	1.75	2.2	42.0	43.7	3.29	1.82	2.3	40.7	44.4	3.29	2.01	2.4
20	1.98	44.0	44.1	3.45	1.68	2.2	43.5	45.2	3.47	1.73	2.2	42.2	43.8	3.47	1.81	2.3	40.9	44.5	3.47	1.98	2.4
21	2.08	44.2	43.9	3.62	1.66	2.2	43.7	45.4	3.65	1.72	2.2	42.4	44.1	3.65	1.79	2.2	41.2	44.2	3.65	1.94	2.3
22	2.18	44.3	44.1	3.80	1.66	2.2	43.9	45.2	3.83	1.70	2.2	42.5	43.7	3.83	1.76	2.2	41.4	44.1	3.83	1.90	2.3
23	2.29	44.3	44.1	3.98	1.65	2.2	43.9	45.0	4.01	1.69	2.2	42.7	43.8	4.01	1.75	2.2	41.6	44.4	4.00	1.88	2.3
24	2.39	44.0	43.7	4.15	1.67	—[b]	43.7	45.3	4.19	1.72	—[b]	42.5	44.0	4.19	1.77	—[b]	41.4	44.2	4.17	1.90	—[b]

[a]The plate area is 466.1 cm^2; [b]End effects are not included in the data reduction or the estimates of the uncertainty, U.

Table 2 Data for a nominal freestream velocity of 67 m/s

		$u_\infty = 67.3$ m/s $T_\infty = 33.8°$C					$u_\infty = 67.4$ m/s $T_\infty = 33.2°$C					$u_\infty = 67.4$ m/s $T_\infty = 33.2°$C					$u_\infty = 67.4$ m/s $T_\infty = 33.1°$C				
P#	$x(m)$	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%	$t_w(C)$	$Q(w)$	Re_x X10^{-6}	St X10^3	U%
1	0.05	38.5	61.2[a]	0.21	3.65	—[b]	32.7	—	—	—	—	32.6	—	—	—	—	32.3	—	—	—	—
2	0.15	40.4	61.2	0.63	2.59	5.7	32.9	—	—	—	—	32.8	—	—	—	—	32.7	—	—	—	—
3	0.25	41.4	61.3	1.05	2.25	4.9	33.1	—	—	—	—	33.0	—	—	—	—	32.9	—	—	—	—
4	0.36	42.0	61.1	1.47	2.05	4.4	33.3	—	—	—	—	33.1	—	—	—	—	32.9	—	—	—	—
5	0.46	42.6	61.2	1.89	1.92	4.1	33.8	—	—	—	—	33.1	—	—	—	—	33.0	—	—	—	—
6	0.56	43.1	61.3	2.31	1.83	3.9	39.3	60.4[a]	2.33	2.69	—[b]	33.2	—	—	—	—	33.0	—	—	—	—
7	0.66	43.3	61.0	2.73	1.78	3.8	40.5	60.3	2.76	2.24	4.7	33.2	—	—	—	—	33.0	—	—	—	—
8	0.76	43.6	61.4	3.15	1.73	3.6	41.2	59.6	3.18	2.07	4.3	33.9	—	—	—	—	33.1	—	—	—	—
9	0.86	43.9	61.1	3.56	1.68	3.5	41.8	60.1	3.61	1.91	4.0	39.5	60.1[a]	3.61	2.61	—[b]	33.1	—	—	—	—
10	0.97	44.0	60.5	3.98	1.64	3.4	42.1	60.4	4.03	1.84	3.9	40.7	59.5	4.03	2.15	4.4	33.2	—	—	—	—
11	1.07	44.2	60.8	4.40	1.62	3.4	42.4	60.3	4.46	1.79	3.7	41.3	59.3	4.46	1.98	4.0	33.2	—	—	—	—
12	1.17	44.4	61.1	4.82	1.60	3.1	42.6	60.3	4.88	1.74	3.6	41.8	59.6	4.88	1.89	3.8	33.2	—	—	—	—
13	1.27	44.5	60.7	5.24	1.57	3.2	42.8	59.5	5.31	1.69	3.6	42.1	59.7	5.33	1.82	3.6	34.0	—	—	—	—
14	1.37	44.7	61.0	5.66	1.55	3.2	43.0	59.6	5.73	1.66	3.5	42.4	59.6	5.73	1.77	3.5	39.3[a]	5.72	2.65	—[b]	
15	1.47	44.8	60.9	6.08	1.54	3.2	43.2	60.2	6.16	1.65	3.4	42.5	59.2	6.16	1.73	3.5	40.9	59.9	6.15	2.11	4.5
16	1.57	44.9	60.9	6.50	1.52	3.1	43.4	60.4	6.58	1.61	3.3	42.9	60.0	6.58	1.69	3.3	41.7	60.9	6.57	1.94	4.0
17	1.68	45.0	61.3	6.92	1.51	3.1	43.5	60.2	7.01	1.59	3.3	42.9	59.4	7.01	1.66	3.3	42.0	60.8	7.00	1.87	3.9
18	1.78	45.1	61.1	7.34	1.50	3.0	43.6	60.2	7.43	1.58	3.3	43.1	59.4	7.43	1.64	3.3	42.2	60.7	7.42	1.81	3.7
19	1.88	45.2	60.9	7.76	1.48	3.0	43.7	60.0	7.86	1.56	3.2	43.2	59.9	7.86	1.62	3.2	42.5	60.8	7.85	1.76	3.6
20	1.98	45.2	61.0	8.18	1.48	3.0	43.8	60.2	8.28	1.55	3.2	43.3	59.3	8.28	1.60	3.2	42.7	60.5	8.27	1.73	3.6
21	2.08	45.4	61.3	8.60	1.47	3.0	43.9	59.9	8.70	1.53	3.2	43.5	59.8	8.70	1.58	3.1	42.9	61.0	8.69	1.70	3.5
22	2.18	45.4	61.1	9.02	1.46	2.9	44.0	60.1	9.13	1.52	3.1	43.6	59.6	9.13	1.56	3.1	43.0	61.0	9.12	1.67	3.4
23	2.29	45.5	61.3	9.44	1.45	2.9	44.1	60.4	9.55	1.51	3.1	43.7	59.8	9.55	1.55	3.1	43.1	60.4	9.54	1.64	3.4
24	2.39	45.3	61.6	9.86	1.46	—[b]	43.9	60.2	9.98	1.53	—[b]	43.5	59.5	9.98	1.57	—[b]	43.0	60.6	9.97	1.67	—[b]

[a]The plate area is 466.1 cm^2; [b]End effects are not included in the data reduction or the estimates of the uncertainty, U.

Figure 2 shows a comparison of the results of the experiments with the integral solution in Eq. (2) and with the finite-difference solutions for $u_\infty = 27$ m/s. Figure 3 shows the same comparison for $u_\infty = 67$ m/s. The results are presented in terms of St/St_t for a direct comparison with Eq. (2). The constant q_w'' data St were normalized with the constant $t_w\,(\phi = 0)$ data, St_t. The constant q_w'' finite-difference solutions were normalized with the constant $t_w\,(\phi = 0)$ finite-difference solutions. Both the St_t data and computations have been previously shown to compare well with the usual correlations.[1,2] As in Eq. (1), the solutions were made on the basis of constant properties evaluated at the freestream static temperature. Buoyancy effects were completely negligible. The figures show that the finite-difference solutions are in very good agreement with the data in all cases. The integral solutions are also in good agreement with the data, with the maximum difference between the data and integral solutions being about 10%. The integral solutions are based on the 1/7 power law velocity and

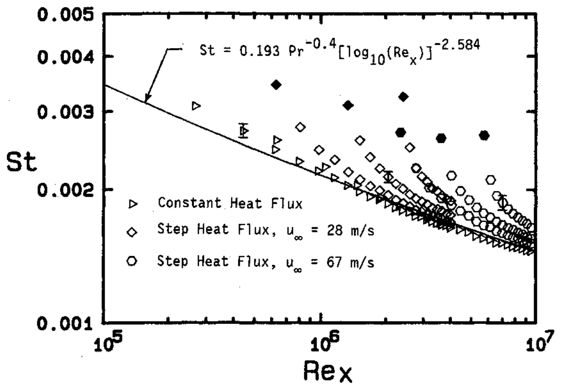

Fig. 1 Summary of the Stanton number data for the constant heat flux boundary condition and the unheated starting length cases.

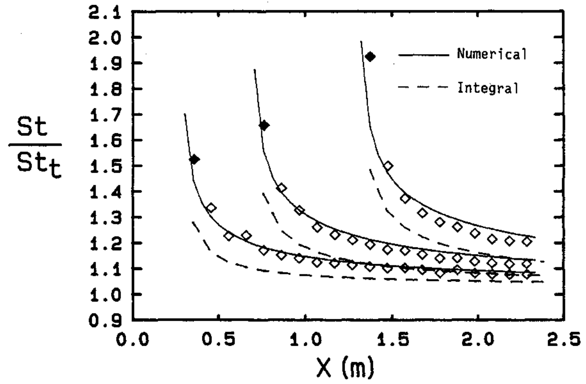

Fig. 2 Comparison of the data with the solutions for $u_\infty = 28$ m/s.

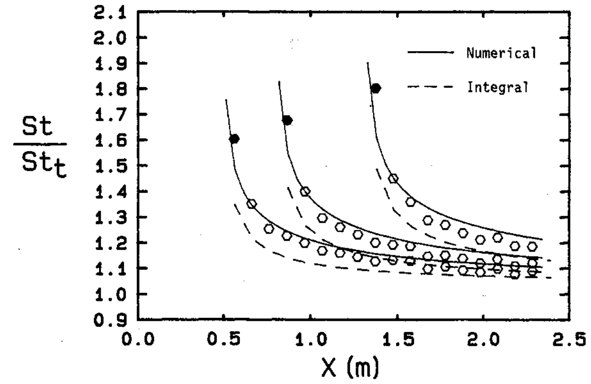

Fig. 3 Comparison of the data with the solutions for $u_\infty = 67$ m/s.

temperature profiles. Therefore, they should be viewed as asymptotic cases in which the boundary layers have become well developed.

References

[1]Reynolds, W. C., Kays, W. M., and Kline, S. J., "Heat Transfer in the Turbulent Incompressible Boundary Layer, Parts I, II, and III," NASA Memo 12-1-58W, 12-2-58W, and 12-3-58W, 1958.

[2]Coleman, H. W., Hosni, M. H., Taylor, R. P., and Brown, G. B., "Smooth Wall Qualification of a Turbulent Heat Transfer Test Facility," TFD-88-2, Mechanical and Nuclear Engineering Department, Mississippi State University, Mississippi State, MS, 1988.

[3]Love, P., Taylor, R. P., Coleman, H. W., and Hosni, M. H., "Effects of Thermal Boundary Condition on Heat Transfer in the Turbulent Incompressible Flat Plate Boundary Layer," Mechanical and Nuclear Engineering Department, Mississippi State University, Mississippi State, MS, TFD-88-3, 1988.

[4]Gatlin, B., "An Instructional Computer Program for Computing the Steady, Compressible, Turbulent Flow of an Arbitrary Fluid Near a Smooth Wall," MS Thesis, Mechanical and Nuclear Engineering Department, Mississippi State University, Mississippi State, MS, 1983.

Thermal Correlation of Natural Convection in Bottom-Cooled Cylindrical Enclosures

Shou-Shing Hsieh*
National Sun Yat-Sen University,
Kaohsiung, Taiwan, Republic of China

Introduction

IN spite of the importance of convective heat transfer in vertical cylindrical enclosures in many practical applications, very few basic studies have been conducted on this system. In Refs. 1-4, one finds that the phenomenon can exist primarily in one of two exteme configurations: 1) a fluid layer heated from below and 2) a fluid layer heated from the side.

Work on the fluid layer heated from top and/or side walls has received rather limited attention as either the experimental study of turbulent natural convection or with rectangular enclosure being the only geometry considered. In fact, the technical application of this study is important to the performance assessment of a solar storage tank. In a series of papers, Yin et al.[5] and Huang[6] experimentally investigated this problem using water and 20 CS Silicone oil as the working media for different aspect ratios of $0.2 \leq H/R \leq 2.0$ and Prandtl numbers of $3 \leq Pr \leq 250$. For numerical study, the related problem of natural convection in a differentially heated corner region of a rectangular enclosure was investigated recently by Kimura and Bejan.[7] Their results show that a unicellular motion exists and migrates toward the corner as the Ra increases.

As stated earlier, the published literature is primarily restricted to the experimental study of either turbulent natural convection or rectangular enclosures. Most recently, Huang and Hsieh[8] investigated the natural convection heat transfer in a cylindrical enclosure cooled from below. There is, however, no reported information on the heat-transfer behavior in cylindrical enclosures of differing aspect ratios. Such a situation is analyzed here. This paper reports on a two-dimensional numerical simulation of buoyance-driven flows, with Prandtl numbers of the working fluid 1, 10, 100, and 200, within vertical cylindrical enclosures of differing aspect ratios (height to radius) of 0.5, 1, and 2 that are cooled from below. It is recognized that the simulation of the present problem is bound to exhibit a certain degree of three dimensionality and unsteadiness. The investigations of Figliola[9] and Kimura and Bejan[7] might be helpful in this regard, providing this bifurcation. With the side wall insulated, the top wall was cooled and the bottom wall heated in the work of Figliola.[9] The resulting flow was two-dimensional until Rayleigh numbers larger than 5×10^4 were imposed, at which point the stable single toroid broke down into three-dimensional motion. For a rectangular

Received Nov. 7, 1988; revision received March 21, 1989.

*Professor, Department of Mechanical Engineering. Member AIAA.